gular bolometer mount may be calibrated from this mount, using the comparison method with a directional coupler and an adaptor [9].

## Conclusion

For a precise power measurement in the millimeter-wave region, an eight-fan-type bolometer mount was designed and tested at a frequency of 100 GHz. This mount has a very high effective efficiency compared to a conventional bolometer mount in the same frequency region. The proposed mount is not only useful for actual power measurement, but is also suitable for calibration of a power standard mount by a calorimeter because of its compact dimension and small heat capacity.

## Acknowledgment

The authors wish to thank K. Yamamura and Dr. T. Iwasaki for their assistance and valuable discussions and also appreciate the encouragement of Dr. K. Sakurai.

## References

[1] A. C. Macpherson and D. M. Kerns, "A microwave microcalorimeter," *Rev. Sci. Instr.*, vol. 26, pp. 27–33, Jan. 1955.
[2] Hitachi mm Wave Test Equipment (Tech. Spec.), Hitachi Ltd., Cat., pp. 30–31, July 1969.
[3] T Nemoto, K. Fujisawa, and T. Inoue, "Newly developed bolometer mount for the short millimeter wave region," *IEEE Trans. Instrum. Meas.*, vol. IM-21, pp. 480–483, Nov. 1972.
[4] K. Sakurai and T. Nemoto, "A thin-film bolometer unit," *IEEE Trans. Instrum. Meas.*, vol. IM-16, pp. 206–211, Sept. 1967.
[5] K. Sakurai and T. Maruyama, "A millimeter wave microcalorimeter," *IRE Trans. Instrum. Meas.*, vol. IM-11, p. 270, Dec. 1962.
[6] T. Inoue and T. Iwasaki, "Unwanted mode generated by a thin-film load in a circular waveguide," in *Nat. Convention Rec. Inst. Electronics and Communication Engineers of Japan*, Microwave Section C, No. 668, Mar. 1976.
[7] B. S. Seip and L. W. Hinderks, "A computerized Klinger cavity mode conversion test set," *IEEE Trans Microwave Theory Tech.*, vol. MTT-22, pp. 873–878, Oct. 1974.
[8] T. Iwasaki and T. Nemoto, "Properties of a thin film loaded in a waveguide," *Bull. Electrotech. Laboratory*, vol. 40, no. 1, pp. 57–62, Jan. 1976.
[9] G. F. Engen, "An evaluation of the 'back-to-back' method of measuring adaptor efficiency," *IEEE Trans. Instrum. Meas*, vol. IM-19, pp. 18–22, Feb. 1976.

# Complex Permittivity of GaAs and CdTe at Microwave Frequencies

WILLIAM E. COURTNEY

*Abstract*—**The microwave dielectric constant and loss tangent of Cr-doped semi-insulating GaAs have been measured in the frequency range 2.5–36.0 GHz and the temperature range 300–400 K. The room temperature dielectric constant is 12.95 and the temperature coefficient $\alpha$ ($\equiv \varepsilon(0)^{-1}\, d\varepsilon/dT$) is $1.6 \times 10^{-4}$/K. The dielectric constant and loss tangent of CdTe have been measured as functions of temperature at 15.95 GHz. The room temperature dielectric constant is $10.39 \pm 0.04$ and the temperature coefficient $\alpha$ is $2.5 \times 10^{-4}$/K.**

## I. Introduction

The COMPLEX permittivity of Cr-doped semi-insulating GaAs at microwave frequencies is of interest both to microwave component designers and the solid-state physicist. In microwave integrated circuits using GaAs devices the material is both the substrate for the epitaxial layer and the microwave transmission medium in the form of a microstrip or slot line. For electrooptic modulation at microwave frequencies the dielectric constant is required for the design of the device and the matching structures, and knowledge of the loss tangent is needed to estimate the insertion loss. The effective mass of electrons and holes in a semiconducting material can be inferred from measurements of the dielectric constant as a function of temperature provided the properties of the lattice dielectric constant are known, i.e., the dielectric properties in the absence of charge carriers. The effective ionic charge can also be deduced from measurements of the "static" and optical dielectric constants.

The last decade has seen a large number of published results on the dielectric properties of GaAs, among them low-frequency capacitance measurements [1], [2], microwave measurements in the frequency range 2.0–70.2 GHz [3]–[11], and infrared transmission and reflection measurements [12]–[14]. The above experiments have produced values of relative dielectric constants which vary from 9.8 to 13.3. The temperature coefficient of the dielectric constant has been measured at low frequencies [2], and at 70.2 GHz [8], [9], while there are only two reported values of the loss tangent, one at 9.4 GHz [5] and one at 60 GHz [15].

In the case of CdTe, the only reported measurements of the dielectric constant are either low-frequency capacitance measurements [2], [16]–[18] or infrared measurements [19], except for Popa [15] who has reported a loss tangent measurement at 60 GHz.

Manuscript received August 2, 1976; revised December 21, 1976. This work was supported by the Department of the Army.
The author is with M.I.T. Lincoln Laboratory, Lexington, MA 02173.

This paper presents the dielectric constant and loss tangent of Cr-doped GaAs as functions of both frequency and temperature, and the dielectric constant and loss tangent of CdTe, at 15.95 GHz, as functions of temperature.

## II. Experimental Procedure

The measurement technique consisted of a resonating dielectric disk shorted at both ends as proposed by Hakki and Coleman [20], and has been described elsewhere [21]. Briefly, the resonating structure is a shorted dielectric waveguide and the characteristic equation is given by

$$\alpha \frac{J_0(\alpha)}{J_1(\alpha)} = -\beta \frac{K_0(\beta)}{K_1(\beta)} \tag{1}$$

for the $TE_{onl}$ modes, where $J_0(\alpha)$,$J_1(\alpha)$ are Bessel functions of the first kind, $K_0(\beta)$,$K_1(\beta)$ are modified Bessel functions of the second kind, and

$$\alpha = \frac{\pi D}{\lambda_0}\left[\varepsilon - \left(\frac{l\lambda_0}{2L}\right)^2\right]^{1/2} \tag{2}$$

$$\beta = \frac{\pi D}{\lambda_0}\left[\left(\frac{l\lambda_0}{2L}\right)^2 - 1\right]^{1/2} \tag{3}$$

where $D$ and $L$ are the diameter and length of the disk, respectively, $\varepsilon$ is the dielectric constant, $\lambda_0$ is the free-space wavelength, and $l$ is the number of longitudinal variations of the field along the disk axis. In this paper the $TE_{011}$ mode is used, and the dielectric constant $\varepsilon$ can be obtained by measuring the resonant frequency of the structure, solving (1) for the first roots, and calculating the dielectric constant using (2) and (3).

The loss tangent is given by [20],

$$\tan \delta = \frac{A}{Q_0} - B \tag{4}$$

where $A$ and $B$ are functions of $\alpha$ and $\beta$ and $Q_0$ is the unloaded $Q$ of the structure. The term $B$ in (4) contains the conductor loss of the structure which cannot be measured directly, and, hence, it is necessary to assume a value of the dc conductivity. By measuring the unloaded $Q$ of the resonating structure and calculating $A$ and $B$—using the values of $\alpha$,$\beta$, the dimensions of the disk, and the skin depth—the loss tangent can be obtained using (4).

To measure the dielectric properties of GaAs as a function of frequency, seven disks of varying dimensions were prepared. Five disks were cut from a single boule of Cr-doped GaAs [22], and the diameters ground until all five disks were circular to within 0.02 mm. Two disks were cut ultrasonically from a second crystal of GaAs [23]. The original dimensions of the seven disks are shown in Table I. The lengths of the disks were then progressively shortened to provide a series of measurements in the frequency range 4.5–36.0 GHz. The dimensions of the single disk of CdTe [24] are also shown in Table I.

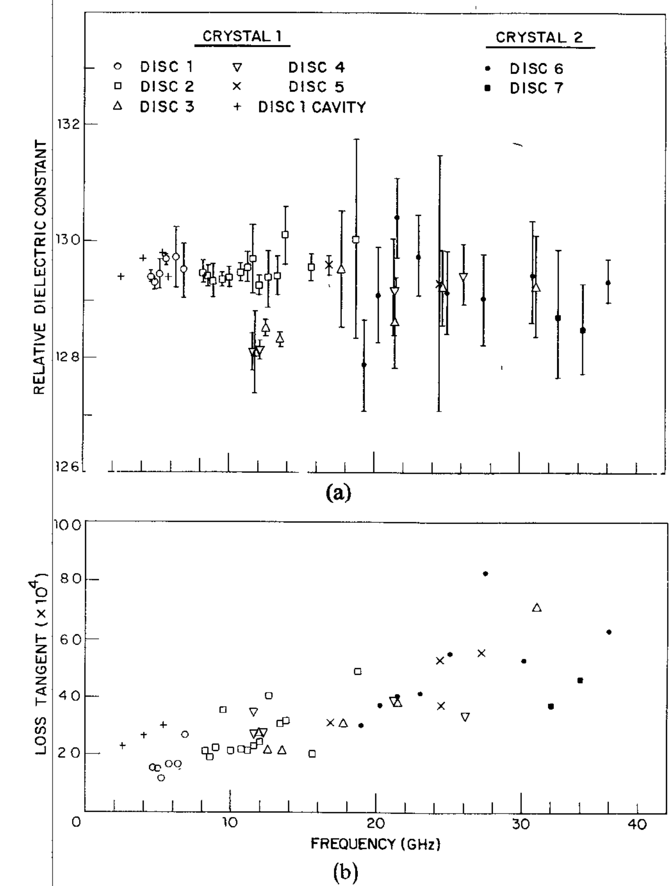


Fig. 1. (a) Relative dielectric constant of Cr-doped GaAs as a function of frequency. (b) Loss tangent of Cr-doped GaAs as a function of frequency.

TABLE I
Original Dimensions of the Dielectric Disks

| | Disc N° | D (mm) | L (mm) |
|---|---|---|---|
| Crystal 1 GaAs | 1 | 25.2425 ± .0076 | 12.6289 ± .0025 |
| | 2 | 12.9134 ± .0076 | 7.6137 ± .0064 |
| | 3 | 9.8610 ± .0041 | 4.9606 ± .0229 |
| | 4 | 9.6828 ± .0025 | 5.0851 ± .0102 |
| | 5 | 6.5583 ± .0025 | 3.5662 ± .0025 |
| Crystal 2 GaAs | 6 | 5.6464 ± .0381 | 3.1750 ± .0102 |
| | 7 | 2.7915 ± .0152 | 2.2479 ± .0025 |
| | CdTe | 7.6200 ± .0178 | 4.2355 ± .00381 |

## III. Results—Cr-Doped GaAs

The dielectric constant and loss tangent as functions of frequency are shown in Fig. 1(a) and (b). The estimate of the error in the measurement of the dielectric constant was obtained by assuming a disk of either the maximum or minimum dimensions and combining these dimensions with the uncertainty in the frequency measurement to give the upper and lower limits. While the above method of calculation of the uncertainty in the dielectric constant does not correspond to the actual physical situation, in which the maximum and minimum lengths of the disks are due to the two surfaces not being parallel, it is believed that this method of calculation gives a conservative estimate of the accuracy.

The accuracy of the loss-tangent measurement is ±10 percent in the measurement of the unloaded $Q$ of the

TABLE II
TEMPERATURE COEFFICIENT OF Cr-DOPED GaAs

| Disc | Frequency (GHz) | ε(0) | $\alpha(10^{-4}\ K^{-1})$ |
|---|---|---|---|
| 1 | 4.5 | 12.36 ± .01 | 1.60 ± .02 |
| 2 | 8.2 | 12.32 ± .04 | 1.71 ± .10 |
| 2 | 11.2 | 12.38 ± .06 | 1.57 ± .16 |
| 3 | 11.6 | 12.20 ± .05 | 1.67 ± .10 |
| 4 | 16.8 | 12.35 ± .10 | 1.60 ± .24 |

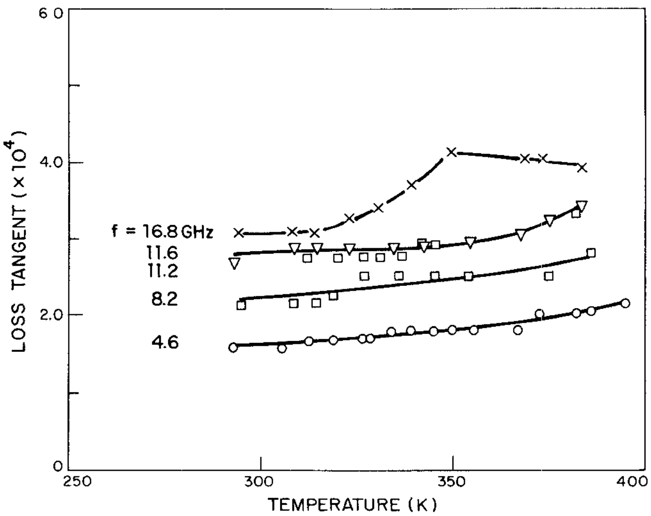


Fig. 2. Loss tangent of Cr-doped GaAs as a function of temperature for measurement frequencies 4.5, 8.2, 11.2, 11.6, and 16.8 GHz, respectively. The symbols are consistent with those used in Fig. 1.

resonator. However, the room-temperature measurements were performed in a laboratory where the relative humidity varied from a low of 35 percent to a high of 68 percent, over the period of the measurements, which is probably the cause of the remaining scatter in the measurements. Besides the above effects, the loss tangent obtained will be high due to the assumption of a bulk dc conductivity in (4). If the microwave conductivity of the Au plating on the shorting plates is 30 percent lower than the bulk dc conductivity, the loss-tangent results will be lower by approximately $0.4 \times 10^{-4}$.

The disk 1 cavity measurements in Fig. 1(a) and (b) were obtained by plating disk 1 with 500 Å of Ti and 5 μm of Au to form a fully filled dielectric cavity with $(D/L) = 3.65$. The four lowest order $TM_{mn0}$ modes were detected and the dielectric constant and loss tangent were calculated using the measured resonant frequencies and unloaded Q-S.

Except for the measurements on disks 3 and 4 (about 12.0 GHz), the results indicate a relative dielectric constant of $12.95 \pm 0.05$ over the frequency range 2.5–36.0 GHz and a loss tangent which varies from $2.0 \times 10^{-4}$ at 2.5 GHz to $6.0 \times 10^{-4}$ at 36.0 GHz.

The temperature coefficient and the loss tangent as a function of temperature were measured by inserting the complete cavity structure in a temperature-stabilized oven. Semirigid 0.085-diameter cable in and out of the oven provided little heat conduction from the structure. The variation of the dielectric constant as a function of temperature can be described by the equation

$$\varepsilon(T) = \varepsilon(0)[1 + \alpha T]. \quad (5)$$

The results of five measurements are shown in Table II, where the values have been corrected for thermal expansion. The loss tangent as a function of temperature is shown in Fig. 2.

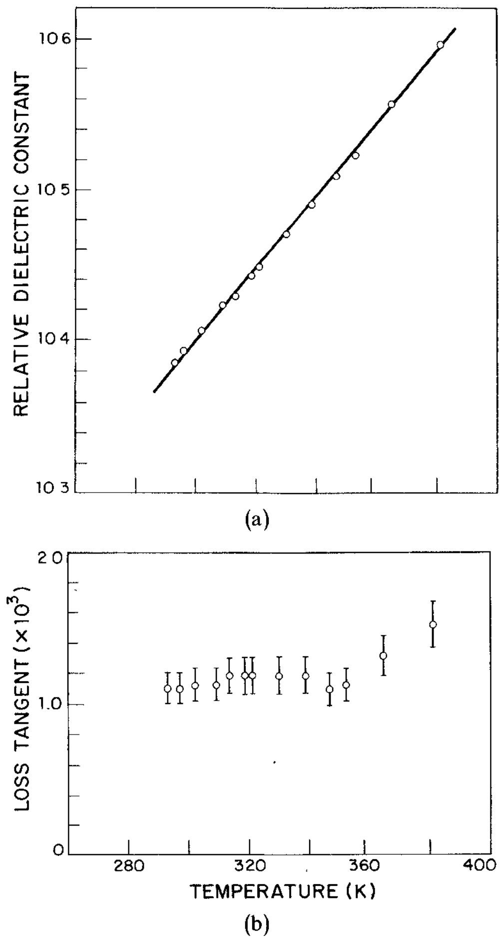


Fig. 3. (a) Relative dielectric constant of CdTe as a function of temperature. (b) Loss tangent of CdTe as a function of temperature.

## IV. RESULTS—CdTe

The disk of CdTe resonated at 15.95 GHz at room temperature giving a value of $10.39 \pm 0.04$ for the relative dielectric constant and $1.1 \times 10^{-3}$ for the loss tangent at 296

TABLE III
PUBLISHED RESULTS OF DIELECTRIC PROPERTIES OF GaAs

| Frequency | ε(0) | ε(296 K) | $\alpha (10^{-4}/ K)$ | Loss Tangent | Ref. |
|---|---|---|---|---|---|
| 1 MHz | | 12.53 ± .10 | | | 1 |
| 20 Hz - 1 MHz | 12.35 ± .07 | 13.08 | 2.01 ± .02 | | 2 |
| 9.4 GHz | | 12.35 ± .10 | | $5.0 \times 10^{-4}$ | 5 |
| 8.57 GHz | | 13.30 ± .50 | | | 7 |
| 9.80 GHz | | 12.90 ± .50 | | | 7 |
| 11.80 GHz | | 12.70 ± .50 | | | 7 |
| 24.52 GHz | | 13.00 ± .50 | | | 7 |
| 46.65 GHz | | 13.14 ± .20 | | | 7 |
| 70.12 GHz | | 12.95 ± .10 | | | 7 |
| 70.24 GHz | 12.73 ± .07 | 13.18 ± .07 | 1.2 ± .10 | | 8 |
| 70.24 GHz | 12.79 ± .10 | | 1.0 | | 9 |
| Infra-red | | 12.80 ± .50 | | | 13 |
| Infra-red | | 13.05 | | | 14 |
| Infra-red | | 13.13 | | | 14 |

K. The dielectric constant and loss tangent as functions of temperature are shown in Fig. 3(a) and (b), where again the correction for thermal expansion has been applied. Referring to (5) the value of $\varepsilon(0)$ is $9.67 \pm 0.08$ and the temperature coefficient $\alpha = 2.5 \pm 0.03 \times 10^{-4}\ K^{-1}$.

## V. DISCUSSION

As a comparison with the results presented here, Table III contains values of $\varepsilon(0)$, $\alpha$, $\varepsilon(296\ K)$, and loss tangents which have appeared in the literature. The values of $\varepsilon(0)$ shown in Table II agree quite closely with the value obtained by Strazalkowski *et al.* [2], although they are somewhat lower than those obtained by Champlin *et al.* [8] and Lu *et al.* [9]. A value of $12.95 \pm 0.10$ for $\varepsilon(296\ K)$ appears to encompass most of the measurements in Table III except those of Hambleton *et al.* [1] and Rogers *et al.* [5]. The measured results for the temperature coefficient $\alpha$ lie between those of Strazalkowski *et al.* [2] and Champlin *et al.* [8] and Lu *et al.* [9]. The measured loss tangent at 9.4 GHz is rather lower than the value obtained by Rogers *et al.* [5], while an extrapolated value of approximately $8.0 \times 10^{-5}$ is higher than the value of $5 \times 10^{-4}$ reported by Popa [15] for 60 GHz.

Strazalkowski *et al.* [2] report values of $\varepsilon(0) = 10.31 \pm 0.08$, $\varepsilon(296\ K) = 11.0$, and $\alpha = 2.27 \pm 0.02$ for CdTe at a frequency of 1 MHz; these values are slightly higher than those reported here. Berlincourt *et al.* [17] quote $\varepsilon(77\ K) = 9.65 \pm 2$ percent at 10 kHz; Yamada [18] measured $\varepsilon(296\ K) = 10.3 \pm 0.2$ at 1 MHz; and Lorimor and Spitzer's infrared measurements [19] gave $\varepsilon(296\ K) = 10.60 \pm 0.15$. The measured loss tangent of $1.1 \times 10^{-3}$ at 15.95 GHz compares with Popa's result [15] of $1.5 \times 10^{-3}$ at 60 GHz.

## ACKNOWLEDGMENT

The author wishes to thank R. W. Sudbury for suggesting the need for the above measurements, C. Bozler for providing the GaAs crystals, G. M. Carter for providing the CdTe disk, R. Tompkins and S. Duda for technical assistance, and the Charles Stark Draper Laboratory, Cambridge, MA, for the use of their equipment to measure the roundness of the disks.

## REFERENCES

[1] K. G. Hambleton, C. Hilsum, and B. R. Holeman, "Determination of the effective ionic charge of gallium arsenide from direct measurements of the dielectric constant," *Proc. Phys. Soc.* (London), vol. 77, pp. 1147–1148, 1961.
[2] I. Strazalkowski, S. Joshi, and C. R. Crowell, "Dielectric constant and its temperature dependence for GaAs, CdTe and ZnSe," *App. Phys. Letts.*, vol. 28, pp. 350–352, Mar. 1976.
[3] S. Jones and S. Mao, "The dielectric constant of GaAs at microwave and millimeter wave frequencies," *App. Phys. Letts.*, vol. 11, pp. 351–353, Dec. 1967.
[4] T. E. Walsh, "Gallium-arsenide electro-optic modulators," *RCA Review*, vol. 27, pp. 323–336, Sept. 1966.
[5] C. B. Rogers, G. B. H. Thompson, and G. R. Antell, "Dielectric constant and loss tangent of semi-insulating GaAs at microwave frequencies," *App. Phys. Letts.*, vol. 11, pp. 353–354, Dec. 1967.
[6] N. Brazlau, "On the dielectric constant of GaAs at microwave frequencies," *App. Phys. Letts.*, vol. 11, pp. 350–351, Dec. 1967.
[7] K. S. Champlin, R. J. Erlandson, G. H. Glover, P. S. Hauge, and T. Lu, "Search for resonance behavior in the microwave dielectric constant of GaAs," *App. Phys. Letts.*, vol. 11, pp. 348–349, Dec. 1967.
[8] K. S. Champlin and G. H. Glover, "Temperature dependence of the microwave dielectric constant of the GaAs lattice," *App. Phys. Letts.*, vol. 12, pp. 231–232, Apr. 1968.
[9] T. Lu, G. H. Glover, and K. S. Champlin, "Microwave permittivity of the GaAs lattice at temperatures between 100°K and 600°K," *App. Phys. Letts.*, vol. 13, p. 404, Dec. 1969.
[10] A. Kumar and P. C. Kothari, "Microwave conductivity and permittivity in germanium, silicon, and gallium arsenide," *Indian. J. Pure and App. Phys.*, vol. 10, pp. 740–741, Oct. 1972.
[11] R. D. Larrabee and W. A. Hicinbothem, "Resonance behavior of the dielectric constant of GaAs at microwave frequencies," *App. Phys. Letts.*, vol. 10, pp. 334–336, June 1967.
[12] B. W. Henvis and M. Hass, "Infra-red lattice reflection spectra of III-V compound semi-conductors," *J. Phys. Chem. Solids*, vol. 23, pp. 1099–1104, 1962.
[13] C. J. Johnson, G. H. Sherman, and R. Weil, "Far infra-red measurement of the dielectric properties of GaAs and CdTe at 300K and 8K,"

App. Optics, vol. 8, pp. 1667–1671, Aug. 1969.

[14] S. Iwasa, I. Balslev, and E. Burstein, "The fundamental infra-red lattice vibration spectra of GaAs," *Proc. 7th Int. Conf. Phys. of Semiconductors* (Paris), pp. 1077–1083. (Academic Press, Dunod, 1969.)

[15] A. E. Popa, "Materials study for millimeter subcarrier optical modulators," *IEEE Microwave Theory Tech.* (Int. Symp. 1973), pp. 295–296, 1973.

[16] D. De Nobel, "Phase equilibria and semiconducting properties of cadmium telluride," *Philips Res. Reps.*, vol. 14, pp. 361–399, Jan. 1959.

[17] D. Berlincourt, H. Jaffe, and L. R. Shiozawa, "Electroelastic properties of the sulfides, selenides, and tellurides of zinc and cadmium," *Phy. Rev.*, vol. 129, pp. 1009–1017, Feb. 1963.

[18] S. Yamada, "On the electrical and optical properties of p-type cadmium telluride crystals," *J. Phys. Soc. Japan*, vol. 15, pp. 1940–1944, Nov. 1960.

[19] O. G. Lorimor and W. G. Spitzer, "Infrared refractive index and absorption of InAs and CdTe," *J. App. Phys.*, vol. 36, pp. 1841–1844, June 1965.

[20] B. W. Hakki and P. D. Coleman, "A dielectric resonator method of measuring inductive capacities in the millimeter range," *IRE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 402–410, July 1960.

[21] W. E. Courtney, "Analysis and evaluation of a method of measuring the complex permittivity and permeability of microwave insulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 476–485, Aug. 1970.

[22] Crystal No. 2453, Crystal Specialties, Inc., Monravia, CA.

[23] Crystal No. 4560, Laser Diode Laboratories, Inc., Metuchen, NJ.

[24] Crystal No. 593, II–VI, Inc., Glenshaw, PA.

# Precise Calibration of Plane-Wave Microwave Power Density Using Power Equation Techniques

HOWARD I. BASSEN AND WILLIAM A. HERMAN

*Abstract*—**A power-density calibration methodology utilizing an anechoic chamber, high-power transmitter, and truncated pyramidal horn antenna is described. Plane-wave power density is accurately computed in the far field of the antenna, based upon precise measurements of antenna gain, absolute transmitted power, and multipath reflections. The application of power equation techniques enables direct precise measurements of system mismatches and the accurate transfer of calibration of special bolometers. Several considerations, unique to hazard probe calibrations, are discussed. Absolute power density uncertainties are estimated at 0.56 dB, at 2450 MHz, and 0.76 dB, at 915 MHz, under worst case conditions. A discussion of second-order error sources and their elimination includes the effects of antenna alignment, antenna sidelobes, multipath reflections, field curvature at noninfinite distances, and scattering from test apparatus.**

## I. Introduction

A PLANE-WAVE power-density calibration methodology has been developed. It permits the accurate far-field calibration of hazard probes which are used to measure leakage from microwave ovens, diathermy machines, and other electronic products emitting radiation in the ISM (industrial, scientific, and medical) frequency bands centered at 915 MHz and 2450 MHz. In addition, these probes are extensively utilized to evaluate the levels present in near- and far-field exposure systems used by biological effects researchers. While a few papers have dealt with the problem of hazard probe calibration [1], [2], there is no rigorous treatment of the overall calibration procedure. In particular, little is written dealing with the accurate transfer of primary electrical power standards to the absolute calibration of radiated power-density levels. Therefore, an overall power-density calibration methodology, which deals with direct traceability to a primary electrical standard has been developed, with particular emphasis on the minimization of cumulative errors or uncertainties.

A large body of literature exists, which deals, both analytically and experimentally, with the determination of the gain of standard horn antennas. Additional problems arise when an electrically small antenna such as a hazard probe is introduced into the overall measurement procedure. These problems are not dealt with in most of the previously available literature, and are, therefore, emphasized in this paper. Only plane-wave far-field power-density calibration procedures are developed in this paper. This is a primary factor in the accuracy of all power-density measurement devices. The many near-field measurement problems, unique to specific leakage probes and their inherent antenna/sensor design are not dealt with in this paper, but are discussed elsewhere [3]. This paper describes a technique for the generation of an accurately defined, traceable, and absolute plane-wave power-density level. This enables far-field calibrations to be performed on a wide variety of instruments with a minimum of uncertainty. Extensive use is made of the power equation techniques, developed by Dr. G. Engen of the National Bureau of Standards [4], allowing the

Manuscript received May 25, 1976; revised January 18, 1977.

The authors are with the Electromagnetics Branch, Division of Electronic Products, Bureau of Radiological Health, U.S. Food and Drug Administration, Rockville, MD 20857.